Mathematics Formula Sheet

1. Read the questions carefully
2. Don't spend too long on one question
3. Show all your working clearly

Algebra

Suitable for Years 7–10

Equations

$6x + 9 = 21$

Subtract both sides by 9:

$6x = 12$

Divide both sides by 6:

$x = 2$

Grouping Terms

$4x - 7x = -3x$

$-2y^2 + 7x - 4y^2 = -6y^2 + 7x$

Expansion

$a(b + c) = ab + ac$

$(a + b)(c + d)$

$= ac + ad + bc + bd$

Binomial expansion: use Pascal's triangle to help expand $(a+b)^n$.

Factorisation

Common factor:

$16x^3 - 8x^2 + 2x$

$= 2x(8x^2 - 4x + 1)$

Difference of two squares:

$a^2 - b^2 = (a + b)(a - b)$

Guessing method:

$x^2 - 7x + 12 = (x - 3)(x - 4)$

Radicals

$\sqrt{a}\sqrt{a} = a$

$\sqrt{ab} = \sqrt{a}\sqrt{b}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Simplifying radicals:

$\sqrt{28} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

$\sqrt{200} = \sqrt{100}\sqrt{2} = 10\sqrt{2}$

Rationalising the denominator:

$$\frac{4}{3-\sqrt{7}} = \frac{4}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}}$$

Index Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$(ab)^m = a^m b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$a^{-m} = \frac{1}{a^m}$

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

$a^{\frac{1}{m}} = \sqrt[m]{a}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Recursion

Calculating the next term from the previous term.

$a_n = a_{n-1} + 3$

Simultaneous Equations

$2x + 3y = 7$ ①

$3x - 7y = -1$ ②

Make coefficients of x the same:

$6x + 9y = 21$ ③

$6x - 14y = -2$ ④

③ minus ④ to cancel x:

$23y = 23$

$y = 1$

Substitute $y = 1$ into ①:

$2x + 3(1) = 7$

$x = 2$

Compound Interest

$P(1 + r)^n$

Logarithms

$\log_a(xy) = \log_a(x) + \log_a(y)$

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

$\log_a(x^n) = n\log_a(x)$

Changing the base:

$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$

Re-write in exponential form:

$\log_a(x) = y \quad \Longrightarrow \quad x = a^y$

Sequences & Series

Arithmetic sequence:

$a_n = a_1 + (n - 1)d$

Arithmetic series:

$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$

Geometric sequence:

$a_n = a_1 r^{n-1}$

Geometric series:

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

Infinite geometric series:

$S_\infty = \frac{a_1}{1 - r}$

Imaginary Numbers

$i = \sqrt{-1}$

$i^2 = -1$

Types of Numbers

$\mathbb{N}$: Natural num. $\mathbb{Z}$: Integers

$\mathbb{Q}$: Rational num. $\mathbb{R}$: Real num.

Combinatorics

$^nC_r = \frac{n!}{r!(n-r)!}$

$^nP_r = \frac{n!}{(n-r)!}$

Graphs

Suitable for Years 7–10

Linear Graphs

To sketch a linear graph, plot two points and connect them.

To find the equation of a graph:

1 Start with $y = mx + c$.

2 Find m:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

3 Substitute point and solve c.

Given a line $y = 2x + 3$:

Parallel line has $m = 2$

Perpendicular line has $m = \frac{-1}{2}$

Quadratic Graphs

For all forms, find x-intercepts and vertex.

Tips for different forms:

1 $y = ax^2 + bx + c$

Use quadratic formula to find x-intercepts.

2 $y = a(x - b)(x - c)$

Use null factor law to find x-intercepts.

3 $y = a(x - h)^2 + k$

Vertex is (h, k)

Use a to determine if parabola opens upwards or downwards:

Quadratic Formula

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

$b^2 - 4ac < 0 \Longrightarrow$ no solutions

$b^2 - 4ac = 0 \Longrightarrow$ 1 solution

$b^2 - 4ac > 0 \Longrightarrow$ 2 solutions

Null Factor Law

For $(x + 5)(2x + 7) = 0$,

$x + 5 = 0 \Longrightarrow x = -5$

$2x + 7 = 0 \Longrightarrow x = \frac{-7}{2}$

Intercepts

Finding y-intercepts:

Substitute $x = 0$ and solve for y.

Finding x-intercepts:

Substitute $y = 0$ and solve for x.

Mid-point and Distance

Midpoint:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance:

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Parent Functions

$f(x) = x$ $\quad f(x) = x^2$

$f(x) = x^3$ $\quad f(x) = |x|$

$f(x) = \frac{1}{x}$ $\quad f(x) = \frac{1}{x^2}$

$f(x) = \sqrt{x}$ $\quad f(x) = e^x$

$f(x) = \log_e(x)$ $\quad f(x) = \sin(x)$

$f(x) = \cos(x)$ $\quad f(x) = \tan(x)$

Graph Transformations

Move left c: $x \to x + c$

Move right c: $x \to x - c$

Move up c: $y \to y - c$

Move down c: $y \to y + c$

Reflect over x-axis: $y \to -y$

Reflect over y-axis: $x \to -x$

Horizontal compression: $x \to 2x$

Horizontal stretch: $x \to \frac{x}{2}$

Vertical compression: $y \to 2y$

Vertical stretch: $y \to \frac{y}{2}$

Domain/Range

Domain: all values of x

Range: all values of y

$y = \sqrt{x} \Longrightarrow x \geq 0$

$y = \frac{1}{x} \Longrightarrow x \neq 0$

Composite Functions

$f(x) = x + 1$

$g(x) = x^2$

$fg(x) = f(g(x))$

$= f(x^2)$

$= x^2 + 1$

Inverse Functions

① Swap x with $f(x)$

② Replace $f(x)$ with $f^{-1}(x)$

③ Make $f^{-1}(x)$ the subject

Graphing: reflect over $y = x$

Domain of $f(x)$ = Range of $f^{-1}(x)$

Range of $f(x)$ = Domain of $f^{-1}(x)$

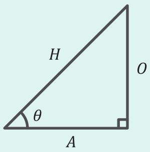
Trigonometry
Suitable for Years 7–10

Trigonometry
For right-angled triangles:

$\sin(\theta) = \frac{O}{H}$

$\cos(\theta) = \frac{A}{H}$

$\tan(\theta) = \frac{O}{A}$

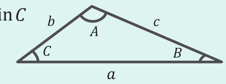


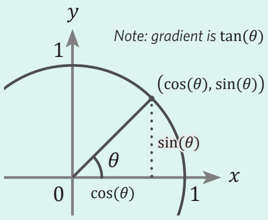
For any triangle:

$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

$Area = \frac{1}{2}ab\sin C$

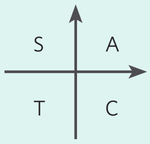


Unit Circle


Radians to degrees:

$2\pi = 360°$

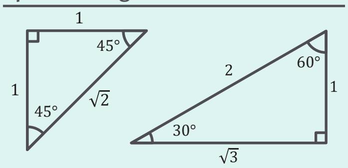
$\pi = 180°$



Trigonometric Identities
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

Special Triangles


Exact Value Table
	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	—

Shapes & Geometry
Suitable for Years 7–10

Volume
Cylinder: base area × height

Pyramid/Cone: $\frac{1}{3}$× base area × height

Sphere: $\frac{4}{3}\pi r^3$

Surface Area
Cylinder: $2\pi r^2 + 2\pi rh$

Cone: $\pi rs + \pi r^2$

Sphere: $4\pi r^2$

Interior/Exterior Angles
Angles of a triangle add to 180°.

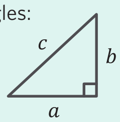
For an n-sided polygon:

Interior angles: $(n-2) \times 180°$

Exterior angles: 360°

Pythagoras
For right-angled triangles:

$c^2 = a^2 + b^2$



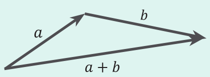
Congruent Triangles
SSS: side, side, side

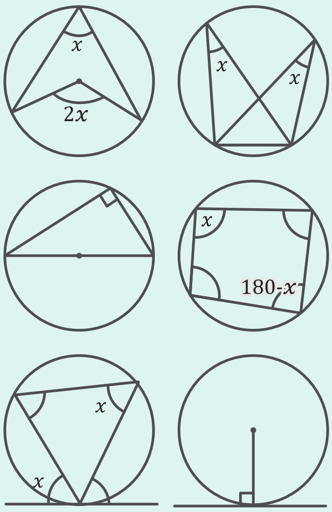
SAS: side, angle, side

ASA: angle, side, angle

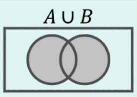
AAS: angle, angle, side

RHS: right-angle, hypotenuse, side

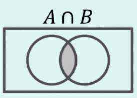
Vectors


Circle Theorems


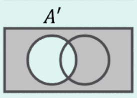
Set Theory
Union:
Inside A or B.



Intersection:
Inside A and B.



Complement:
Outside of A.



Conditional Probability
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Calculus
Suitable for Years 11–12

Limits
$$\lim_{x \to 3} \frac{(x-3)(x-2)}{x-3}$$

Keep simplifying since substituting $x = 3$ makes the denominator 0.

$$\lim_{x \to 3}(x-2) = 3 - 2 = 1$$

Differentiation by First Principles
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Differentiation
Differentiation is used to find the slope of a graph.

$$\frac{d}{dx}(8x^2 - 3x + 5) = 16x - 3$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

Chain Rule
$$\frac{d}{dx}(f(u)) = f'(u) \times u'$$

Product Rule
$$\frac{d}{dx}(uv) = uv' + u'v$$

Quotient Rule
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

Integration
Integration is the opposite of differentiation and is often used to find the area under a graph.

Indefinite integrals:

$$\int 4x - 1\,dx = 2x^2 - x + c$$

Note: remember to add "+c".

Definite integrals:

$$\int_1^3 x + 2\,dx$$

$$= \left[\frac{1}{2}x^2 + 2x\right]_1^3$$

$$= \frac{1}{2}(3)^2 + 2(3) - \left(\frac{1}{2}(1)^2 + 2(1)\right)$$

$$= 8$$

Note: don't need to add "+c".

Integration by Parts
$$\int uv'\,dx = uv - \int vu'\,dx$$