

Mark Scheme

Summer 2023

Pearson Edexcel GCE In Mathematics (9MA0) Paper 01 Pure Mathematics

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- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 100.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- ***** The answer is printed on the paper
- The second mark is dependent on gaining the first mark

- 4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 5. Where a candidate has made multiple responses <u>and indicates which response</u> <u>they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most</u> <u>complete</u>.

- 6. Ignore wrong working or incorrect statements following a correct answer.
- 7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q)$$
, where $|pq| = |c|$, leading to $x = ...$
 $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = ...$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Scheme Marks					
1	$x^{\frac{1}{2}}(2x-5) = \dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}} \text{ or } \frac{x^{\frac{1}{2}}(2x-5)}{3} = \frac{\dots x^{\frac{3}{2}} + \dots x^{\frac{1}{2}}}{3}$	M1	1.1b		
	$\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3}$	A1	1.1b		
$\int \frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3} dx = \dots x^{\frac{5}{2}} \pm \dots x^{\frac{3}{2}} (+c) dM1$					
	$\frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$	A1	1.1b		
		(4)			
	Notos		(4 marks)		
M1·	Notes Attempts to multiply out the brackets of the numerator and either writes the	vnressio	n (or just the		
1011.	Autompts to multiply out the blackets of the numerator and entiter writes the		$\frac{1}{2}$ $\frac{1}{2}$		
	numerator) as a sum of terms with indices . Award for either one correct indecomes from a correct method. Condone appearing as terms on separate lines	ex of $\dots x^2$ s for this	$x^2 + \dots x^2$ which mark. The		
	correct index may be implied later when e.g. $\sqrt{x} \rightarrow x^{\frac{3}{2}}$ or $(\sqrt{x})^3 \rightarrow x^{\frac{3}{2}}$ after	er they ha	ave integrated.		
	The $\frac{1}{3}$ does not need to be considered for this mark.				
A1:	$\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3} \text{ or equivalent e.g. } \frac{1}{3} \left(2x^{\frac{3}{2}} - 5x^{\frac{1}{2}} \right). \text{ Condone } \frac{2x^{\frac{3}{2}} - 5x^{\frac{1}{2}}}{3} \text{ May be}$	implied 1	by further work.		
	The correct index may be implied later when e.g. $\sqrt{x} \rightarrow x^{\frac{3}{2}}$ or $(\sqrt{x})^3 \rightarrow$	$x^{\frac{5}{2}}$ after	r they have		
	Integrated. Ignore incorrect integration notation around the terms. Ignore any presence	or absenc	the of dx .		
	Be aware that a factor of $\frac{1}{2}$ may be taken outside of the integral so you b	may need	l to look at		
	further work to award the first A mark if work on the two terms is done. May be unsimplified and the two terms may appear in a list which is fine. Coefficients must be exact.	e separat	ely or in a list.		
dM1:	Increases the power by one on an x^n term where <i>n</i> is a fraction. The index d processed.	oes not n	eed to be		
	e.g. $x^{\frac{3}{2}+1}$ or $x^{\frac{1}{2}+1}$ It is dependent on the previous method mark so at least have had a correct index.	one of th	e terms must		
	Note that integrating the numerator and denominator e.g. $\frac{2x^{\frac{3}{2}}}{3} - \frac{5x^{\frac{1}{2}}}{3} \rightarrow {3}$	$\frac{x^{\frac{5}{2}}}{3x} - \frac{x^{\frac{3}{2}}}{3x}$	- is dM0.		

 $\frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$ and including the constant or simplified exact equivalent such as A1: $\frac{4}{15}\sqrt{x^5} - \frac{10}{9}\sqrt{x^3} + c \text{ or } \frac{4}{15}x^{2.5} - \frac{10}{9}x^{1.5} + c \text{ or } \frac{1}{45}\left(12x^{\frac{5}{2}} - 50x^{\frac{3}{2}}\right) + c \text{ or } \frac{x^{\frac{7}{2}}}{45}(12x - 50) + c.$ Fractions must be in their lowest terms and indices processed. Do not accept e.g. $0.267x^{\frac{5}{2}} - 1.11x^{\frac{3}{2}} + c$ but allow $0.26x^{\frac{5}{2}} - 1.1x^{\frac{3}{2}} + c$ Isw once a correct answer is seen but withhold this mark if there is spurious notation around their final answer e.g. $\int \frac{4}{15} x^{\frac{5}{2}} - \frac{10}{9} x^{\frac{3}{2}} + c \, dx$ is M1A1dM1A0 Alternative method using integration by parts example e.g. $\int x^{\frac{1}{2}} (2x-5) dx = \dots x^{\frac{3}{2}} (2x-5) - \int \dots x^{\frac{3}{2}} dx$ (applies integration by parts correctly to typically M1: achieve this form – the (2x-5) may also be split up as well – send to review if unsure how to mark) This may also be done the other way round e.g. $\int x^{\frac{1}{2}} (2x-5) dx = \dots x^{\frac{1}{2}} (x^2-5x) - \int \dots x^{\frac{3}{2}} \pm \dots x^{\frac{1}{2}} dx$ The $\frac{1}{2}$ does not need to be considered for this mark. A1: A correct intermediate stage applying integration by parts with correct coefficients. e.g. $\left(\frac{x^2(2x-5)}{3} dx = \frac{2}{3}x^{\frac{3}{2}}\left(\frac{2x-5}{3}\right) - \int \frac{4}{9}x^{\frac{3}{2}} dx$ (or unsimplified equivalent). Coefficients must be exact. (See main scheme notes above) The other way round this could appear as e.g. $\left(\frac{x^{\frac{1}{2}}(2x-5)}{3} dx = \frac{1}{3}x^{\frac{1}{2}}(x^2-5x) - \frac{1}{6}\int x^{\frac{3}{2}} - 5x^{\frac{1}{2}} dx$. Condone a missing dx. May be implied. Increases the power by one on an x^n term where *n* is a fraction e.g. $\int ...x^{\frac{3}{2}} dx \rightarrow ...x^{\frac{5}{2}}$ The index dM1: does not need to be processed. It is dependent on the previous method mark. $\frac{4}{15}x^{\frac{3}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c \text{ or exact simplified equivalent. (See main scheme notes above)}$ A1: Alternative method using the substitution method e.g. let $u = x^{\frac{1}{2}} \Rightarrow \int ...u^4 + ...u^2 du$ (uses a substitution to express the integral in terms of another M1: variable. Allow slips with the coefficients, but the indices should be correct for their substitution) The $\frac{1}{2}$ does not need to be considered for this mark. e.g. $\int \frac{4u^4}{3} - \frac{10u^2}{3} du$ or unsimplified equivalent. Coefficients must be exact. See main scheme A1: notes above). May be implied by further work. Condone a missing dx. $\int ...u^4 + ...u^2 du \rightarrow ...u^5 + ...u^3$ (increases the power by one on at least one of their indices – does not dM1: need to be processed. It is dependent on the previous method mark. $\frac{4}{15}x^{\frac{5}{2}} - \frac{10}{9}x^{\frac{3}{2}} + c$ or exact simplified equivalent. (See main scheme notes above) A1: There may be alternative substitutions, but the same marking principles apply.

Question	Scheme	Marks	AOs			
2(a)	$(f(a) =) 4a^3 + 5a^2 - 10a + 4a = 0 \Longrightarrow a() = 0$	M1	3.1a			
	$a(4a^2+5a-6)=0$ *	A1*	1.1b			
		(2)				
(b)(i)	$a = \frac{3}{4}$	B1	2.2a			
(ii)	$4x^{3} + 5x^{2} - 10x + 4 \times "\frac{3}{4}" = 3 \Longrightarrow 4x^{3} + 5x^{2} - 10x (= 0)$	M1	1.1b			
	x = 0	B1	1.1b			
	$x = \frac{-5 \pm \sqrt{185}}{8}$	A1	1.1b			
		(4)				
		(6	marks)			
(a)	Trotes					
A1*: Ao M If Do More diff	A1*: Achieves the given answer with no errors including brackets. Minimum acceptable is $4a^3 + 5a^2 - 10a + 4a = 0 \Rightarrow a(4a^2 + 5a - 6) = 0$ If the = 0 is absent at the start of their solution, it must appear before achieving the given answer. Do not allow attempts to find the value of <i>a</i> and substitute that into $f(x)$ More difficult alternative methods may be seen:					
$4x^2$	$+(5+4a)x + (-10+5a+4a^2)$					
$x-a \overline{)4x^3}$	$+5x^2$ $-10x$ $+4a$					
$4x^{3}$	$-4ax^2$					
	$(5+4a)x^2$ -10x	- - 2	4 3 0			
	$(5+4a)x^2$ $-a(5+4a)x$ Then sets remainder	$r -6a + 5a^2 + 6a^2 +$	$4a^{3} = 0$			
	$(-10+5a+4a^2)x + 4a$					
	$(-10+5a+4a^2)x - a(-10+5a+4a^2)$					
	$-6a+5a^2+4a^3$					
M1: Fo	or dividing the cubic by $(x-a)$ leading to a quadratic quotient in x and a cub hich is then set = 0 and attempts to take a factor of a out.	ic remainder i	n <i>a</i>			
Al*: Co	completely correct with $a(4a^2 + 5a - 6) = 0$					

Alt 2: You may also see a grid or an attempt at factorisation via inspection OR $4x^3 + 5x^2 - 10x + 4a \equiv (x - a)(4x^2 + px - 4)$ which should be followed by equating the x terms and x^2 terms to form two equations which can be solved simultaneously. -10 = -ap - 4 and $5 = -4a + p \Longrightarrow p = 5 + 4a$ $\Rightarrow -10 = -a(5+4a) - 4 \Rightarrow 4a^2 + 5a - 6 = 0 \Rightarrow a(4a^2 + 5a - 6) = 0$ The above are examples. There may be other correct attempts so look at what is done. M1: For an attempt to set up two simultaneous equations by equating coefficients for x and equating coefficients for x^2 . Condone slips. $4a^2 + 5a - 6 = 0 \Longrightarrow a(4a^2 + 5a - 6) = 0$ Completely correct with no errors. A1*: (b) Mark (i) and (ii) together (i) Deduces that $a = \frac{3}{4}$ only. May be implied by their resultant cubic. If they do (b)(ii) multiple times B1: using other roots for which $a \neq \frac{3}{4}$, then the solutions arising from using the other roots $a \neq \frac{3}{4}$ must be rejected (ii) Attempts to substitute their $a = \frac{3}{4}$ (which must be positive) into f(x), sets their f(x) = 3 and M1: collects terms on one side (= 0 may be implied). Condone arithmetical and sign slips. Condone if they repeat this step using their other root(s). B1: (x =) 0 $(x =) \frac{-5 \pm \sqrt{185}}{2}$ (and these values only) or exact equivalent (ignore 0 for this mark). Withhold this A1: mark if the fraction line was clearly not intended to be under both terms. This mark cannot be scored if they proceed directly to the roots from $4x^3 + 5x^2 - 10x$ without taking a factor of or dividing by x first to see the quadratic factor. Isw once the correct answers are seen if they proceed to provide rounded answers after. $4x^3 + 5x^2 - 10x + 4 \times \frac{3}{4} = 3 \Longrightarrow 0, \frac{-5 \pm \sqrt{185}}{9}$ is M0B1A0 e.g. $4x^3 + 5x^2 - 10x = 0 \Longrightarrow 0, \frac{-5 \pm \sqrt{185}}{8}$ is M1B1A0 e.g. e.g. $4x^3 + 5x^2 - 10x = 0 \Rightarrow 4x^2 + 5x - 10 = 0 \Rightarrow \frac{-5 \pm \sqrt{185}}{8}$ is M1B0A1 e.g. $4x^3 + 5x^2 - 10x = 0 \Rightarrow x(4x^2 + 5x - 10) = 0 \Rightarrow 0, \frac{-5 \pm \sqrt{185}}{8}$ is M1B1A1

QuestionSchemeMarksAOs3(a)
$$(\overline{(OA|} =) \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38} *$$
B1*1.1b(b) $\overline{(OB|} = \sqrt{2^2 + 4^2 + a^2} = \sqrt{20 + a^2}$ so when $a = 5$ $\overline{OB|} = \sqrt{20 + 25} = \sqrt{45}$ M11.1b $= 5$ Alcso2.3(2)(3 marks)(a) $\overline{OB} = \sqrt{2^2 + 4^2 + a^2} = \sqrt{20 + a^2}$ so when $a = 5$ $\overline{OB} = \sqrt{20 + 25} = \sqrt{45}$ M11.1b $= 5$ Alcso2.3(2)(3 marks)(a) $\overline{OA} = 5^2 + 3^2 + 2^2 = 38 \Rightarrow \overline{(OA|} = \sqrt{5^2 + 3^2 + 2^2}$ or e.g. $\sqrt{25 + 9 + 4}$. We need to seehow the value 38 or $\sqrt{38}$ is formed using the three components. Withhold this mark for incorrectworking such as $|\overline{OA|} = 5^2 + 3^2 + 2^2 = 38 \Rightarrow \overline{(OA|} = \sqrt{15} + 3^2 + 2^2$ as to what they are finding (OA instead of $\overline{(OA|}$ is fine). Do not penalise if their square root does not go fully over all three terms as long as the intention is clear as to what they are finding (OA instead of $\overline{(OA|}^2 =) 25 + 9 + 4 = 38 \Rightarrow (\overline{OA|} =) \sqrt{38}$ scores B1 (we see how 38 is found) $(\overline{(OA|}^2 =) 25 + 9 + 4 = 38 \Rightarrow (\overline{OA|} =) \sqrt{38}$ scores B1 (we see how 38 is found) $25 + 9 + 4 = \sqrt{38}$ scores B0 (hey are not equal) $(\overline{(OA|}^2 =) 38 \Rightarrow (\overline{OA|} =) \sqrt{38}$ scores B0 (no method scen to show how 38 is found)(b)M1:Attempts to find $|\overline{OB|}|$ or $\overline{OB|}^2$ in terms of a and substitutes in a positive integer for a to find a valuefor $|\overline{OB|}|$ or $|\overline{OB|}|$ or $\overline{OB|}^2$ in terms of a and substitutes in a positive integer for a to find a valuefor $|\overline{OB|}|$ or $\overline{OB|}^2$ in terms of a and substitutes in a positive integer for a to find a valuefor $|\overline{OA|}| =$

Question Scheme Mark				
4a	$2\alpha + \frac{1}{2}\left(1 - \frac{\alpha^2}{2}\right)$	M1	1.2	
	$2\alpha + \frac{1}{2}\left(1 - \frac{\alpha^2}{2}\right) = 0 \Longrightarrow 2\alpha + \frac{1}{2} - \frac{\alpha^2}{4} = 0 \implies \alpha = \dots$	dM1	1.1b	
	$\alpha = -0.243 \text{ (3dp) only}$	A1	2.3	
h	1	(3)		
D	$f'(0) = \frac{1}{2}\cos 0 \Longrightarrow \Longrightarrow y =x + 3$	M1	1.1b	
	$y = \frac{1}{2}x + 3$	A1	1.1b	
		(2)		
	NT-4	(5 n	narks)	
(a)	Note on EPEN this is M1A1A1 but we are marking this as M1dM1A1			
(u)	Accept to be in terms of α or another variable e.g. r			
	Note: 0.242 with no working is 0 works			
	Note: -0.243 with no working is 0 marks			
M1:	Fully substitutes $\cos x = 1 - \frac{x^2}{2}$ into the derivative.			
dM1:	: Attempts to multiply out to achieve a $3TQ (= 0)$ and attempts to find a value for α . Condone slips. Allow solving the quadratic via any method (usual rules apply).			
	If they use a calculator then you may need to check this.			
A1:	$(\alpha =) -0.243$ only cao Can only be scored provided a correct 3TQ is seen. If both re	oots found t	then	
	the other one must be rejected (or a choice made of -0.243 e.g. underlining it or a t	ick)		
	Condone $x = -0.243$			
(b)				
M1:	Attempts to find the gradient of the curve when $x = 0$ and achieves an equation of the $y = f'(0) + 3$.	he form		
	x = 0 must be fully substituted in and a value must be found for the gradient. Do not if they attempt to use a changed gradient e.g. the gradient of the normal.	ot allow this	s mark	
	Also allow attempts using the small angle approximation:			
	$f'(x) \approx 2x + \frac{1}{2} \left(1 - \frac{x^2}{2} \right)$ when $x = 0$, $f'(0) = "\frac{1}{2}" \Longrightarrow y = "f'(0)"x + 3$			
A1:	$y = \frac{1}{2}x + 3$ or equivalent in the form $y = mx + c$ isw Stating just the values $m = 0.5$	c = 3 with	out	
	the correct equation is A0			

Quest	ion Scheme	Marks	AOs
5(a	h = 0.2	B1	1.1b
	$\frac{1}{2} \times "0.2" \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59$	M1	1.1b
	e.g. $\Rightarrow a+13.5+2b+111.4=175.9 \Rightarrow a+2b=51*$	A1*	2.1
		(3)	
(b)	$a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2 \Rightarrow a + b = 28 \Rightarrow a = \dots (\text{ or } b = \dots)$	M1	3.1a
	a=5 or b=23	A1	1.1b
	a = 5 and $b = 23$	A1	1.1b
		(3)	
	NI - 4	(6	o marks)
(a)	Notes		
B1: M1:	States or uses $h = 0.2$ o.e. Forms the equation $\frac{1}{2} \times "0.2" \times \{a + 13.5 + 2(16.8 + b + 20.2 + 18.7)\} = 17.59$ o.e.	but condone of	copying
	slips. They may have added some of the y values together so as a minimum acc " 0.1 "×{ $a+13.5+2(55.7+b)$ } = 17.59	ept e.g.	
	achieving the given answer. Condone the use of \approx for this mark. Allow this mark if they add the areas of individual trapezia e.g. $\frac{\text{their }h}{2}\{a+16.8\} + \frac{\text{their }h}{2}\{16.8+b\} + \frac{\text{their }h}{2}\{b+20.2\} + \frac{\text{their }h}{2}\{20.2+18.7\} + \frac{1000}{2}\{20.2+18.7\} + \frac$	$\frac{\text{their }h}{2} \{18.7 + h \text{ must be num}\}$	-13.5} merical
A1*:	A rigorous argument leading to $a + 2b = 51$ from correct working and no errors brackets, although do not penalise a missing trailing bracket at the end e.g.	seen includir	ng
	$\frac{1}{2} \times "0.2" \times \{a+13.5+2(16.8+b+20.2+18.7)=17.59 \Longrightarrow \Rightarrow a+2b=51 \text{ could}$	score B1M1A	Albut
	$\frac{1}{2} \times "0.2" \times a + 13.5 + 2(16.8 + b + 20.2 + 18.7) = 17.59 \Longrightarrow \dots \Longrightarrow a + 2b = 51 \text{ could s}$	core max B11	M1A0
	provided later work implied correct brackets. Both sets of brackets must be dealt with correctly before proceeding to the final e.g. $ \Rightarrow a + 2b + 124.9 = 175.9 \Rightarrow a + 2b = 51$ is M1A1* $ \Rightarrow a + 13.5 + 33.6 + 2b + 40.4 + 37.4 = 175.9 \Rightarrow a + 2b = 51$ is M1A1* $ \Rightarrow 0.1a + 1.35 + 3.36 + 0.2b + 4.04 + 3.74 = 17.59 \Rightarrow a + 2b = 51$ is M1 Note that $a + 2b \approx 51$ as the final answer is A0*	answer such A1*	that
(b) M1:	Attempts to form the equation $a+16.8+b+20.2+18.7+13.5=97.2$, condonin (may just be stated as e.g. $a+b=28$ o.e.) and attempts to solve their equation is the given equation (or condone their equation from part (a)). Do not be too com- process here as calculators may be used. Score if values for <i>a</i> or <i>b</i> are reached f simultaneous equations.	g copying err imultaneousl cerned with th rom a pair of	ors, y with ne
Al:	tor $a = 5$ or $b = 23$		

A1: for both a = 5 and b = 23

Quest	ion Scheme	Marks	AOs	
6(a) $\frac{1}{2}a$	B1	2.2a	
		(1)		
(b)	$\log_2 x(x+8) \Longrightarrow \log_2 x + \log_2(x+8)$	M1	1.2	
	=a+b	A1	2.2a	
		(2)		
(c)	e.g. $8 + \frac{64}{x} = \frac{8x + 64}{x}$	B1	1.1b	
	$\log_2 \frac{8}{x}(x+8) = 3 - \log_2 x + \log_2 (x+8)$	M1	1.1b	
	3+b-a	A1	2.2a	
		(3)		
	Notos	(6	marks)	
Conde	notes	eview.		
(a)	, ne omission of Suse 2 in an pares. If energy work in any other Suse then send to r			
B1:	$\frac{1}{a}$ or $\frac{a}{b}$ or 0.5 <i>a</i> isw			
ക	2 2			
(D) M1:	Takes a factor of x out of the bracket to achieve $\log_2 x(x+8)$ and attempts to apply	y the additi	ion law	
	of logs usually leading to $\log_2 x + \log_2 (x + 8)$ Condone missing brackets or omission of base 2			
	May be implied by a correct answer. Allow this mark to be scored if they write $\log_2 x + \log_2 x + \log_2 8$ (an answer of $2a + 3$ can score M1A0)			
	$\log_2 x \times \log_2 (x+8)$ on its own is M0 but allow the mark to be scored if they proceed	ed to $a+b$		
A1:	a+b or simplified equivalent (a correct answer with no incorrect log work seen scores M1A1) isw Note $\log_2 x \times \log_2 (x+8) = a+b$ is M1A0 (allow the answer to imply the correct method but withhold the final work)			
(c)				
B1:	Writes $8 + \frac{64}{x}$ as a single fraction e.g. $\frac{8x+64}{x}$ or $\frac{8}{x}(x+8)$ or $8x^{-1}(x+8)$ or $8\left(\frac{2}{x}\right)$	$\left(\frac{x^2+8x}{x^2}\right)w$	hich	
	may be implied by later work e.g. $\log_2 8 - \log_2 x + \log_2 (x+8)$			
M1:	Attempts to apply the laws of logs, uses $\log_2 8 = 3$ and proceeds to $3 \pm \log_2 x \pm \log_2 x$	$_{2}(x+8)$		
	(or equivalent since $\pm \log_2 x$ may appear as $\pm \log_2 \frac{1}{x}$ or $\pm \log_2 x^{-1}$)			
	May be implied by $3\pm b\pm a$ and condone invisible brackets around $x+8$ and condomission of base 2.	one the		
	Note that if they write $\log_2(x+8)$ as $\log_2 x + \log_2 8$ this is M0			
A1:	3+b-a or simplified equivalent (a correct answer with no incorrect log work see	n is B1M1	A1) isw	
	Note $\log_2 \frac{8}{x}(x+8) = 3 \div \log_2 x \times \log_2(x+8) \Longrightarrow 3 - a + b$ is B1M1A0 (allow the answ	ver to imp	ly the	
Note: answe	correct method but withhold the final mark) You may see attempts to work backwards to the answer. If these do not result is r but you think they are creditworthy then send to review.	in the cori	rect	

Questi	on Scheme	Marks	AOs	
7(a)	f(x) > 33	B1	1.1b	
		(1)		
(b)	$y = 3 + \sqrt{x - 2} \Longrightarrow x = \dots$	M1	1.1b	
	$f^{-1}(x) = (x-3)^2 + 2$	A1	1.1b	
	x > 33	B1ft	2.2a	
		(3)		
(c)	$f(6) = 3 + \sqrt{6-2} = 5 \Longrightarrow g("5") = \frac{15}{"5"-3} = \dots$	M1	1.1b	
	$=\frac{15}{2}$	A1	1.1b	
		(2)		
(d)	$3 + \sqrt{a^2 + 2 - 2} = \frac{15}{a - 3} \Longrightarrow "a^2 - 9 = 15"$	M1	1.1b	
	$a = 2\sqrt{6}$	A1	2.2a	
		(2)		
			(8 marks)	
(a)	Notes			
(a) B1:	f(x) > 330.e.			
	e.g. $y > 3$, range > 3, $f(x) \in (3, \infty)$, $\{f(x) : f(x) > 3\}$, $f > 3$ but not e.g. x	>3, $f(x)$	33 [3, ∞)	
M1:	Sets $y = 3 + \sqrt{x-2}$ and attempts to make <i>x</i> the subject (or vice versa). Look operations so score for an expression of the form $(x=)$ $(y \pm 3)^2 \pm 2$ or $(y=)$ $f^{-1}(x) = (x-3)^2 + 2$ Also accept $f^{-1}: x \rightarrow (x-3)^2 + 2$. Condone $f^{-1} = (x-3)^2 + 2$	for the co $(x\pm 3)^2 \pm (x\pm 3)^2$	orrect order of	
	$f^{-1} = y = (x-3)^2 + 2$) but do not allow just $y =$ or $f^{-1} : y =$			
	so accept other equivalent expressions such as $f^{-1}(x) = x^2 - 6x + 11$ (simplified or unsimplified)			
B1ft:	$x > 33$ or follow through on their part (a). The omission of $x \in \Box$ is condon Allow equivalent answers such as $x \in ("3", \infty)$ or $x \in [x; x > "3"]$	ed.		
Note:	Allow equivalent answers such as $x \in (-3^{+}, \infty)$ or $\{x : x > -3^{+}\}$ It is also acceptable to define f^{-1} in any variable e.g. as $f^{-1}(t) = (t-3)^{2} + 2$ $t > 33$ long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $f^{-1}(t) =$ not just $f^{-1} =$			
(c) M1.	Substitutes $x = 6$ into f and substitutes the result into a to find a value for	af(6)		
1011.	Substitutes $x = 0$ into 1 and substitutes the result into g to find a value for 15	gr (0) .	4	
	Allow an attempt to substitute $x = 6$ into $gf(x) = \frac{1}{\sqrt{x-2}}$ condoning slips.	They mus	t proceed to	
	find a value. Condone arithmetical slips and bracket errors/omissions. Cond	one for M	1 attempts	
	where when dealing with $\sqrt{x-2}$ leads to two different answers e.g. $\frac{15}{\sqrt{6-2}}$	$\rightarrow \pm \frac{13}{2}$		
A1:	$\frac{15}{2}$ only oe is once a correct answer is seen			

(d) Attempts to form the equation $3 + \sqrt{a^2 + 2 - 2} = \frac{15}{a - 3}$, and proceeds to a quadratic in *a* (usually M1: $a^2 = k$ or $a^2 - k = 15$ but condone arithmetical, miscopying and sign slips. Condone equations which would lead to complex roots. May be implied by a correct exact answer. Alternatively, they attempt to form the equation $a^2 + 2 = f^{-1}g(a) \Rightarrow a^2 + 2 = \left(\frac{15}{a-3} - 3\right)^2 + 2$ \Rightarrow (a+3)(a-3)=15 \Rightarrow a²-9=15 (condone slips) They should be square rooting both sides so that $\sqrt{a^2 + 2 - 2} \rightarrow a$, before multiplying both sides by a-3 and rearranging so that the a^2 term comes from their "(a+3)(a-3)" May be implied by a correct exact answer for their quadratic in a but a correct decimal answer does not imply this mark. $(a =) 2\sqrt{6}$ or accept $\sqrt{24}$ (they must reject the negative solution if found as $f(a^2 + 2) \neq g(a)$ when A1: $a = -2\sqrt{6}$) $\sqrt{6} \times \sqrt{4}$ is A0 isw $\sqrt{24}$ followed by $4\sqrt{6}$ (incorrect manipulation of the surd) but not followed by $\pm\sqrt{24}$ o.e.

A decimal answer on its own or multiple answers e.g. $\pm \sqrt{24}$ score A0.

Questi	on Scheme	Marks	AOs	
8(a)	<i>OC</i> ×2.3=27.6	M1	1.1b	
	e.g. $OC = \frac{27.6}{2.3} = 12 \mathrm{m}^{-8}$	A1*	2.1	
		(2)		
(0)	e.g. $(2AOB =) \pi - 2.3$	M1	1.1b	
	$\frac{\pi - 2.3}{2} \Rightarrow 0.421 \text{ rad } *$	A1*	2.1	
(c)	1	(2)		
(1)	Area $OCDE = \frac{1}{2} \times 12^2 \times 2.3$	M1	1.1b	
	$= 165.6 (\text{m}^2) (\text{accept awrt } 166)$	A1	1.1b	
	$(OB =) \frac{35 - 27.6}{2} + 12 = 15.7 \text{ m}$	B1	2.1	
	Area of <i>OAB</i> (or <i>OFG</i>) = $\frac{1}{2}$ × "15.7"×7.5×sin 0.421 (= 24.0m ²)	M1	1.1b	
	Total area = $165.6 + 2 \times 24.1$	dM1	3.1a	
	$= awrt \ 214 (m^2)$	A1	1.1b	
		(6)		
	Notos	(10) marks)	
M1: Uses $l = r\theta$ with $l = 27.6$ and $\theta = 2.3$ correctly substituted in (may be labelled differently in their equation). Values just need to be embedded in an equation or accept an expression for OC e.g. $\frac{27.6}{2.3}$. May work in degrees which is acceptable. Condone an alternative letter being used to denote OC such as r Alternatively, they use $l = r\theta$ with $r = 12$ and $\theta = 2.3$ and verify that $l = 27.6$ m A1*: Achieves an expression for OC before proceeding to $OC = 12$ (m) with no errors seen (condone lack of units)				
	$r = \frac{27.6}{2.3} = 12 \text{ is M1A1* (condone alternative letters for } OC)}$ BUT e.g. $\frac{27.6}{2.3} = 12(\text{m}) \text{ on its own is M1A0*}$			
e.g. $OC \times 2.3 = 27.6 \Rightarrow OC = 12(m)$ is M1A0* In the alternative method they verify $l = 27.6$ and conclude that $OC = 12 m$ We must see the calculation $12 \times 2.3 = 27.6$ and conclude that $OC = 12(m)$ e.g. $arc = 12 \times 2.3 = 27.6$ so $OC = 12(m)$ is M1A1* whereas $12 \times 2.3 = 27.6$ is M1A0*				
	If they work in degrees and use rounded values this scores A0* (If they work with	e.g. $\frac{414}{\pi}$ 1	to keep	

the angle exact then A1* can still be scored)

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radians. e.g. $\frac{\pi - 2.3}{2} = 0.421$ (rad) is M1A1* e.g. $\frac{180 - \text{awrt} 131.8}{2} \div \frac{180}{\pi} = 0.421$ (rad) is M1A1* (conversion from degrees to radians seen) e.g. $\pi - 2.3 \div 2 = 0.421$ (rad) is M1A0* (invisible/lack of brackets) e.g. $\pi - 2.3 = \frac{0.842...}{2} = 0.421$ M1A0* (incorrect joined statement)

(c)

(b) M1:

A1*:

Attempts to use $A = \frac{1}{2}r^2\theta$ with r = 12 and $\theta = 2.3$ The values embedded in the formula is M1: sufficient for this mark. May be implied by a correct answer or further work. Look out for alternative

more complex ways to find the area of the sector. e.g. area of semicircle - area of two sectors with r = 12 and $\theta = 0.421$

- A1: awrt 166 (may be implied by later work)
- B1: A correct expression or value for the length OB or OF which may be a part of a calculation (may see 15.7 in the equation to find the area of *AOB*)
- Attempts to find the area of at least one of the two congruent triangles using their OB found from M1: $\frac{35-27.6}{2}+12 (=15.7), OA = 7.5 \text{ and } \theta = 0.421 \text{ in } \frac{1}{2} \times OA \times OB \times \sin C \text{ (may work in degrees)}$

Be aware that omitting sine in the formula may give a value close to the area of the triangle which would be M0. Condone use of $\theta = 0.4$ or $\theta = 0.42$ if they have rounded angle AOB.

The values embedded in the expression is sufficient to score the mark or may be implied by the value.

Look out for more complex methods to find the area of one or both of the two congruent triangles

e.g. they may split the congruent triangle into two right angled triangles and add the separate areas.



Ques	tion Scheme	Marks	AOs	
9(:	a) $12-3k k+16$	N / 1	2.1	
	$\overline{3k+4} = \overline{12-3k}$	M1	3.1a	
	$3k^2 - 62k + 40 = 0$ *	A1*	1.1b	
		(2)		
(b)	(i) $3k^2 - 62k + 40 = 0 \Longrightarrow k = \dots$	M1	1.1b	
	States $k = 20$ and gives a reason e.g. that this gives a values of r such th $ r < 1$	at A1	3.2a	
(ii	a = 64 and $r = -\frac{3}{4}$ (or allow $a = 6$ and $r = \frac{5}{3}$)	B1	1.1b	
	$S_{\infty} = \frac{"64"}{1 - "\left(-\frac{3}{4}\right)"} = \dots$	M1	3.1a	
	$S_{\infty} = \frac{256}{7}$	A1	1.1b	
		(5)		
		(7	marks)	
work. Possible equations below (which are not exhaustive) should use <i>n</i> th term or sum of series formulae e.g. $\frac{12-3k}{3k+4} = \frac{k+16}{12-3k}$ or $\left(\frac{12-3k}{3k+4}\right)^2 = \frac{k+16}{3k+4}$ or $(12-3k)^2 = (3k+4)(k+16)$ or $(3k+4)\left(\frac{k+16}{12-3k}\right) = 12-3k$ or $(12-3k)\left(\frac{12-3k}{k+16}\right) = k+16$ or $3k+4+12-3k+k+16 = \frac{(3k+4)\left(1-\left(\frac{k+16}{12-3k}\right)^3\right)}{1-\frac{k+16}{12-3k}}$ (sum of three terms)				
A1*: Achieves the given quadratic with no errors including invisible brackets. It cannot be for just proceeding in one step from the starting equation to the given answer and usually will involve attempting to multiply out brackets or dealing with any fractions.				
M1:	M1: Attempts to solve the given quadratic achieving at least one value for k. Usual rules apply for solving a quadratic and this may be achieved directly from a calculator. (May also be implied by $\frac{2}{3}$)			

20 and gives correct reasoning (if *r* is found anywhere in part (i) then it must be correct): A1: e.g. 20 since |r| < 1. e.g. since |r| = 0.75 < 1e.g. by listing at least two consecutive terms for k = 20 (must be correct) e.g. 64, -48 do not withhold this mark if they proceed to make a comment e.g. "the numbers are getting smaller" as we are condoning this to mean they are referring to the magnitude of the numbers e.g. when k = 20, $r = -\frac{3}{4}$ o.e. which is between 1 and -1 (condone "it is smaller than 1"). Do not accept a reason on its own which is just simply stating that the sequence is converging or equivalent such as "spiralling". Allow reasoning which excludes $k = \frac{2}{3}$ e.g. $r = \frac{5}{3}$ which is greater than 1. (ii) Work may be seen in part (i), but must be used in part (ii) to score. a = 64 and $r = -\frac{3}{4}$ o.e. (or allow a = 6 and $r = \frac{5}{3}$ o.e.) May be implied by later work or a correct B1: calculation using these values to find S_{∞} A full attempt to find S_{∞} by using their value of k to reach a value for r such that |r| < 1 and a value M1: for a. Condone sign slips in their calculations of a and r only. You may need to check this by substituting in their value for *k* if no calculations are seen. They must substitute these values in to $\frac{a}{1-r}$ correctly so e.g. a = 64, $r = -\frac{3}{4} \Rightarrow S_{\infty} = \frac{64}{1-\frac{3}{4}}$ is M0. They cannot just substitute in their k as r in the formula. Do not allow attempts to manually calculate the values of lots of terms for this mark as this would not lead to the answer. $\sum_{n=1}^{\infty} 64 \times \left(-\frac{3}{4}\right)^{n-1}$ on its own is M0. $\frac{256}{7}$ cao. (Do not allow 36.6 as this is not S_{∞}) isw after a correct exact answer is seen. A1:

Quest	tion	Scheme	Marks	AOs	
10(a))(i)	Centre $(-3k, k)$	B1	2.2a	
(11))	$(x+3k)^{2}-9k^{2}+(y-k)^{2}-k^{2}+7=0 \Longrightarrow (x+3k)^{2}+(y-k)^{2}=$	M1	1.1b	
		Radius $\sqrt{10k^2-7}$	A1ft	2.2a	
			(3)		
(b))	$x^{2} + (2x-1)^{2} + 6kx - 2k(2x-1) + 7 = 0 \Longrightarrow \dots x^{2} + (pk+q)x + rk + s(=0)$	M1	1.1a	
		$5x^{2} + (2k-4)x + (2k+8) (=0)$	A1	1.1b	
		$(2k-4)^2 - 4 \times 5 \times (2k+8) = 0 \Longrightarrow k = \dots$	dM1	2.1	
		Critical values = $7 \pm \sqrt{85}$	A1	1.1b	
		$k < "7 - \sqrt{85}"$ or $k > "7 + \sqrt{85}"$ o.e.	ddM1	3.1a	
		$k < 7 - \sqrt{85}$ or $k > 7 + \sqrt{85}$ o.e.	A1	2.5	
			(6)		
		Natas	(9	marks)	
(a)(i)		INOLES			
B1:	(-3)	(k, k) o.e. Accept without brackets. May be written as $x = -3k$, $y = k$			
M1: A1ft:	 Altempts to find r² by completing the square and collects terms outside the brackets on the other side of the equation. (x±3k)²k² + (y±k)²k² + 7 = 0 ⇒ (x±3k)² + (y±k)² = ±ak²±b Alternatively, they may try to use general formulae such as x² + y² + 2fx + 2gx + c = 0 ⇒ r² = f² + g² - c May also be implied by an expression for r. A1ft: √10k² - 7 Condone unsimplified equivalent expressions such as √9k² + k² - 7 and do not allow if this is written with the equation of the circle as (x+3k)² + (y-k)² = 10k² - 7. It must be extracted from this and explicitly written as √10k² - 7 o.e. Do not penalise if their square root does not go fully over all three terms as long as the intention is clear. Only follow through on a centre of the form (±3k, ±k) which will lead to a radius of √10k² - 7 Do not allow ±√10k² - 7 and do not isw e.g. if they divide their radius by 2 (thinking they had 				
(b) M1: A1:	Substitutes $y = 2x - 1$ into the equation of the circle or their manipulated equation of the circle from (a) and attempts to collect terms proceeding to $x^2 + (pk+q)x + rk + s = 0$ where p, q, r and s are all non zero. Condone arithmetical slips and do not be too concerned by the mechanics of their rearrangement. May be implied by $5x^2 + (2k-4)x + 2k + 8 (= 0)$ or by their values for a, b and c in their discriminant. Do not be concerned with the use of $<$, $>$ or $=$ $5x^2 + (2k-4)x + 2k + 8 (= 0)$ (which may be implied by their a, b and c in their discriminant) Do not be concerned with the use of $<$, $>$ or $=$ Check carefully the signs of $2k - 4$ since $4 - 2k$ will lead to the same answers and should score				

Attempts to find $b^2 - 4ac$ for their 3TQ and attempts to find at least one critical value. Do not be too dM1: concerned by the mechanics of their rearrangement. If they find the root(s) directly from a calculator you will need to check this. (condone decimals which may be rounded or truncated) It is dependent on the first method mark. Do not be concerned with the use of <, > or = A1: $7\pm\sqrt{85}$ ddM1: Attempts to find the outside region for their critical values. It is dependent on the previous two method marks. (Must have two values to be able to score this mark) States e.g. $k < "7 - \sqrt{85}"$, $k > "7 + \sqrt{85}"$ (condone $k \dots 7 + \sqrt{85}"$, $k \dots 7 - \sqrt{85}"$). Condone for this mark $x \leftrightarrow k$ and e.g. "7+ $\sqrt{85}$ ", k, "7- $\sqrt{85}$ ". Allow any equivalent expression including set notation which includes both outside regions. Do not penalise poor notation to indicate the outside regions. Condone e.g. "and" o.e for this mark. $k < 7 - \sqrt{85}$ or $k > 7 + \sqrt{85}$ or any equivalent expression including set notation which includes **both** A1: outside regions. e.g. $k < 7 - \sqrt{85}$, $k > 7 + \sqrt{85} (-\infty, 7 - \sqrt{85}) \cup (7 + \sqrt{85}, \infty)$ $\left\{k: k \in \Box, k < 7 - \sqrt{85}\right\} \cup \left\{k: k \in \Box, k > 7 + \sqrt{85}\right\}.$ Allow "," "or", " \cup " or a space between the answers (or on different line) but do not accept "and", If a variable is used it must be in terms of k Do not allow e.g. " $k < 7 - \sqrt{85}$ and $k > 7 + \sqrt{85}$ " or $\left[-\infty, 7 - \sqrt{85}\right] \cup \left[7 + \sqrt{85}, \infty\right]$ isw provided there is no contradiction with the correct answer. **Alternative method:** Using the formula for the perpendicular distance of a point from a line via a Further Maths method Send to review if you are unsure how to mark these Substitutes the values of 2x - y - 1 = 0 and (-3k, k) into $d = \left| \frac{2(-3k) + (-1)k + (-1)}{\sqrt{2^2 + (-1)^2}} \right|$ M1: Condone sign slips. $(d =) \left| \frac{2(-3k) + (-1)k + (-1)}{\sqrt{2^2 + (-1)^2}} \right|$ A1: dM1: Attempts to proceed from $\left|\frac{2(-3k) + (-1)k + (-1)}{\sqrt{2^2 + (-1)^2}}\right| < \sqrt{"10k^2 - 7"}$ to form a 3TQ (typically $k^2 - 14k - 36 > 0$) and attempts to find the critical values as above via any valid method. Do not be concerned with the use of <, > or = A1ddM1A1: As above

Question	Scheme	Marks	AOs		
11(a)	$\log_{10} V = 3 \Longrightarrow V = 10^3$	M1	1.1b		
	(V =) £1000	A1	3.4		
		(2)			
(b)	e.g. $(\log_{10} b =) \frac{2.79 - 3}{10 - 0} = -0.021$ or $\log_{10} V = 3 - 0.021t$ or $10^{2.79} = "1000" b^{10}$	M1	1.1b		
	e.g. $b = 10^{-0.021}$ (= 0.952796) or $V = 10^3 \times 10^{-0.021t}$ or $b = \sqrt[10]{"0.61659"}$	M1	3.1b		
	$V = 1000 \times 0.953^{t}$	Alft	3.3		
		(3)			
(c)	e.g. $V = 1000 \times "0.953"^{24} (= \pounds 315)$				
	or (actor)	M1	3.4		
	e.g. $\log_{10} V = 3 - "0.021" \times 24 \Longrightarrow V = (= \pounds 313)$				
	which is close (to $\pounds 320$) so it is a suitable model	A1	3.2b		
		(2)	0.20		
		(7	' marks)		
	Notes				
of 10 ³ or implied by the correct answer. There may be more complicated routes to finding the initial value. e.g. finding a complete equation such as $\left(\log_{10} V = 3 + \frac{2.79 - 3}{10}t \Rightarrow \log_{10} V = 3 \Rightarrow V =\right)10^3$ This mark can also be scored for the equation $V = 10^3 \times 10^{"-0.021"t}$ or $V = 1000 \times ()^t$ but not $V = 10^{3-"0.021"t}$ (the 10 ³ has not been split up from $10^{"-0.021"t}$) A1: £1000 cao (including units) do not accept £10 ³					
(b) Mar	k (b) and (c) together. Note work seen in (a) must be used in (b) to score				
 M1: Either finds the gradient between the two points. Score for the expression 2.79-3/10-0 o.e. e.g0.021 Do not condone sign slips for this mark. May be implied by later work such as sight of 10^{-0.021}. finds the equation for log₁₀ V in terms of t e.g. log₁₀ V = 3-"0.021"t which may be unsimplified. forms the equation 10^{2.79} (= 616.5) = "1000"b¹⁰ o.e. such as 2.79 = 3+10log b 					
M1: A S • th Y • ca	 M1: Attempts to find the value or an expression for <i>b</i> using their gradient or their equation Score for either: the expression 10^{"-0.021"} o.e such as 10^{2.79-3}/₁₀₋₀ or may be implied by a correct value using their gradient. You may need to check this on your calculator. correctly proceeding from log₁₀ V = 3-"0.021"t to V = 10^{3-"0.021"t} and splitting this into 				

 $V = 10^3 \times 10^{-"0.021"t}$

• attempting to equate coefficients:

 $\log_{10} V = \log_{10} a + (\log_{10} b)t \Leftrightarrow \log_{10} V = 3 - "0.021"t \Longrightarrow \log_{10} b = "-0.021" \Longrightarrow b = 10^{"-0.021"}$

• using their equation $10^{2.79} = "1000"b^{10}$ or $2.79 = "3"+10\log b$ and proceeding to e.g. $b = \sqrt[10]{"0.61659..."}$ or $b = 10^{"-0.021"}$

A1ft: Complete correct equation, follow through on their "1000" so score for $V = "1000" \times (awrt \ 0.953)^t$ or accept $V = "10^3" \times (awrt \ 0.953)^t$. Just stating the values of *a* and *b* is A0ft, but if the equation is written in (c) before substituting in t = 24 then this mark can be awarded.

(c) Mark (b) and (c) together

M1: A full and valid attempt to:

- either substitute t = 24 into their model of the form $V = ab^t$ where *a* is positive and finds a value for *V*
- or substitutes t = 24 into their model of the form $\log_{10} V = p + qt$ where p is positive and finds a value for V (if they only proceed as far as $\log_{10} V$ they would also have to find the value of $\log_{10} 320$)
- or substitutes V = 320 into their $V = 1000 \times 0.953^{t}$ o.e. and finds a value for t

(to enable the candidate to compare real life data with that of the model.)

Do not be too concerned with the mechanics of the solution but they must be attempting to find two values which can be compared (e.g. usually 320 and a value for *V*, but they could proceed to find $\log_{10} 320$ and compare with $\log_{10} V = 2.496$ when t = 24, or a value for *t* to compare with t = 24) In cases with no working you will need to check the calculation.

- A1: Compares their awrt £313-£315 with £320 or their awrt t = 23.5 23.7 with t = 24 or $\log_{10} 320 = 2.505...$ with 2.496 and makes a valid conclusion with a reason. For this mark you require:
 - correct calculations (if using percentage error allow this to be rounded to compare awrt £313-£315 with £320 then it will be in the range (1.4, 2.4). For £314.94 this is = awrt 1.6%)
 - a reason such as "the values are close", "the values are similar", "the values are approximately equal". Allow use of "≈". Allow the calculation of the % error as reason.
 - a statement that it is a "good" or "accurate" model or similar wording.

Note: Condone as a minimum e.g. "£314.94 and £320 so good model" (we accept the two values being stated here as a comparison that they are similar)

Do not allow incorrect statements such as the model is incorrect as it does not give £320. Do not allow just "the model gives an underestimate of the true value" (does not comment sufficiently on whether the model is reliable) Do not allow comments suggesting that the model is not reliable.

Note using the full value for b leads to 313.3285724...

Question	Scheme	Marks	AOs
12	$\sin(x+h) - \sin x$	D1	2.1
	h	BI	2.1
	$\sin x \cos h + \cos x \sin h - \sin x$	M1	1.1b
	h	A1	1.1b
	(As $h \to 0$), $\sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \left(\frac{\sin h}{h}\right) \to 0 \times \sin x + 1 \times \cos x$	dM1	2.1
	so $\frac{dy}{dx} = \cos x$ *	A1*	2.5
	N. A	(5 n	narks)
Theory all as	Notes $\frac{1}{2}$		
There is n	It the question allow the use of $n = 0x$ if used consistently correquirement to see "gradient of chord" written down		
	$\sin(x+h) - \sin x = \sin(x+h) - \sin(x+h)$	$-\sin(x-$	<i>h</i>)
BI: Gi	ves the correct fraction such as $\frac{1}{x+h-x}$ or $\frac{-h}{-h}$ or $\frac{-h}{x+h-x}$	2h	— or
si	$\frac{h(x-h)-\sin x}{\cos h}$ Condone invisible brackets. May be implied by $\frac{\sin x \cos h + \cos x \sin x}{\sin x \cos h + \cos x \sin x}$	$\sin h - \sin h$	<u>x</u>
	x-h-x h		
M1: Us	es the compound angle formula for $sin(x \pm h)$ to give $sin x cos h \pm cos x sin h$		
A1: Ac	chieves $\frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$ or equivalent (may be implied by further work).		
Al	low invisible brackets to be recovered.		
dM1: It Co	: It is dependent on both the B and the M marks being awarded. Complete attempt to apply the given limits to the gradient of their chord. They must isolate $\left(\frac{\cos h - 1}{h}\right)$ and replace with 0 and isolate $\left(\frac{\sin h}{h}\right)$ and replace with 1.		
e.g	$x \sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1$		
Ac	cept as a minimum $\sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \left(\frac{\sin h}{h}\right) = \cos x$ (implying the application	n of the li	mits)
If	hey do not fully show $\left(\frac{\cos h - 1}{h}\right)$ and $\left(\frac{\sin h}{h}\right)$ being isolated but proceed from		
e.g	$\frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$ to $0 \times \sin x + \cos x$ (or e.g. $0 + \cos x$) then this can be in	implied a	nd
sco	bre $dM1$		
	$\frac{1 x (\cos h - 1) + \cos x \sin h}{h} = \cos x \text{ is dM0}$		
Co e.g	ndone if limit notation remains within their expression after the limits have been ap $\lim_{h\to 0} (\sin x \times 0 + \cos x \times 1)$	plied.	
Al	ternatively, condone use of the small angle approximations such that $\frac{1}{2}$		
sin	$\frac{hx\cos h + \cos x\sin h - \sin x}{h} \to \frac{-\frac{h^2}{2}\sin x + h\cos x}{h} = -\frac{h}{2}\sin x + \cos x \text{ and replaces } \frac{h}{2}$	with 0	

A1*: Uses correct mathematical language of limiting arguments to show that $\frac{dy}{dx} = \cos x$ with no errors seen. (cso)

We need to see $h \to 0$ at some point in their solution and linking $\frac{dy}{dx}$ with $\cos x$ e.g.

•
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \dots = \lim_{h \to 0} \left(\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right) = \cos x$$

• $\frac{dy}{dx} = \dots = \lim_{h \to 0} \left(-\frac{h}{2} \sin x + \cos x \right) = 0 \times \sin x + \cos x = \cos x$ (using small angle approximations)

•
$$\frac{dy}{dx} = \dots = \frac{\sin x(\cos h - 1) + \cos x \sin h}{h} = \sin x \times 0 + 1 \times \cos x = \cos x \text{ as } h \to 0$$

Condone f'(x) or y' in place of $\frac{dy}{dx}$

Give final A0 for no evidence of limiting arguments:

e.g. when
$$h = 0$$
 $\frac{dy}{dx} = ... = \sin x \left(\frac{\cos h - 1}{h}\right) + \cos x \left(\frac{\sin h}{h}\right) = \sin x \times 0 + \cos x \times 1 = \cos x$ is A0

Do not allow the final A1 for just stating $\frac{\sin h}{h} = 1$ and $\frac{\cos h - 1}{h} = 0$ and attempting to apply these (without seeing e.g. $h \to 0$ at some point in their solution)

If they work in another variable (e.g. θ) then withhold the final mark. If they have mixed variables within some of their statements, then allow recovery but withhold the final mark.

Withhold this mark if there has been incorrect bracketing or invisible brackets when isolating $\sin x (\cos h - 1)$ e.g. $\frac{\sin x \cos h - 1 + \cos x \sin h}{h}$ but accept terms written as e.g. $\sin x \frac{\cos h - 1}{h}$ which do not require brackets. Condone a missing trailing bracket if the intention is clear.

Quest	tion	Scheme									M	arks	A	Ds							
13(a)			a = 60 B1										3.	1b							
						2 =	="60"	"-b(-	$-20)^{2}$	$\Rightarrow b =$	=							N	M 1	3.	.4
							H = 6	0 - 0.	145(<i>t</i>	-20)	2							I	A1	3.	.3
																		((3)		
(b))	Height = 2 m									Ι	B1	3.	.4							
										((1)										
(c))						α=	180	or β	= 31								Ν	M 1	3.	.4
		$H = 29\cos(9t + 180)^\circ + 31$									1	A 1	3.	.3							
												((2)								
(d))	e.g. "The model allows for more than one circuit"										I	B1	3.:	5a						
																		((1)		
																			(7	' mar	:ks)
									Note	S											
(a)	a - 6	() (L -				1 ~ ~ ~	ation	of 11	1	<u>a</u> 1		.11		0	.h+'	44	-1 f		+l	
BI:	u = 0	00 (ma 1)	ay be	seen	in the	ir fina	ii equ	ation	of the	e mod	elor	impii	ea c	by c	ou su	IDSti	tute	ed IC	or a in	the	
M1·	Atten	nnts te	o find	l <i>h</i> hv	subst	itutin	σ in <i>t</i>	= 0	H = T	2 and	their	a and	1 pro	nce	edin	o to	a 1	value	e for h		
	May	be see	en as	two s	imulta	aneou	s equ	ations	s form	ned:	then	u une	• •		cam	5 10	u	, arac	1010	•	
	2 = a	-b(-	$-20)^{2}$	and	60 = 0	a-b(20 - 2	$(20)^2$	proce	eding	to a v	zalue	for	h							
			-)					- /]	p	B	29		101	0				29		2	
A1:	H = 0	$H = 60 - 0.145(t - 20)^2$ or equivalent such as $H = -\frac{25}{200}t^2 + 5.8t + 2$ or $H = 60 - \frac{25}{200}(t - 20)^2$ isw																			
	once $a = 6$	nce a correct equation for the model is seen. Must be in terms of <i>H</i> and <i>t</i> . If they just state $a = 60, b = 0.145$ then A0																			
	A cor	correct answer with no working seen scores full marks.																			
(b) D1.	2 and (and and look of write). This can be seened areas if their we detail in (a) is incoment (1)																				
D1.	2 cao (condone lack of units) i his can be scored even if their model in (a) is incorrect (they make used symmetry to determine this value)						шау														
(c)	nuve		<u>.</u>	letij (•••••	• •	, and	-)												
M1:	$(\alpha = $	$(\alpha =)$ 180 or $(\beta =)$ 31 Condone $(\alpha =) \pi$																			
A1:	$\tilde{H} = 1$	$H = 29\cos(9t + 180)^\circ + 31$ or equivalent e.g. $H = -29\cos(9t) + 31$ is wonce a correct equation for							r												
	the model is seen. Must be in terms of H and t. If they just state $\alpha = 180$ $\beta = 31$ then A0																				
	A correct equation with no working seen scores both marks. Does not require the degree symbol							bol.													
(d)		-					<u> </u>											<u> </u>			
B1:	Score	e for a	i reaso	on wh	ich m	akes	refere	ence to	o any	of											
	• th	the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits",							s",												
	"(• +1-	"cyclical", "periodic", "loops around", "the original model can only go up and down once") the alternative model after 2 minutes the appring will be hack at the start (a g "at 2 minutes U = 2")																			
	 un th 	the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the																			
	- u	height will be negative which cannot happen")																			
	• th	the original model after 2 minutes would not be back at the start																			
	Do no	Do not allow vague responses on their own e.g. "the original model is a parabola"																			
	If cal	f calculations are used then they must be correct using a correct model (allow rounded or truncated)						ed)													
	Look	for a	valid	reasc	on and	l igno	re ref	erenc	e to a	nythi	ng els	e as 1	ong	as	it do	$\frac{1}{2}$	<u>10t</u>	cont	tradict	;	
	t	0	5	10	15	20	25	30	35	40	45	50	55		60	80		100	120	_	
	h	2	27	46	56	60	36	46	27	2	-31	-/1	<u> -11</u>	8 -	1/2	-46	2 -	-868	-139()	

Questic	n Scheme	Marks	AOs		
14	When <i>n</i> is even:				
	$(2k+1)^{3} - (2k)^{3} = 8k^{3} + 12k^{2} + 6k + 1 - 8k^{3} = 6k(2k+1) + 1$	N (1	2.1		
	\Rightarrow which is odd	MI	3.1a		
	When n is odd:				
	$(2k+2)^{3} - (2k+1)^{3} = 8(k^{3}+3k^{2}+3k+1) - (8k^{3}+12k^{2}+6k+1) = 6k(2k+3)+7$	A1	2.2a		
	\Rightarrow which is odd				
	When <i>n</i> is even: $(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3 = 6k(2k+1) + 1$				
	\Rightarrow which is odd	JM1	2.1		
	and When <i>n</i> is odd:	alvii	2.1		
	$(2k+2)^{3} - (2k+1)^{3} = 8(k^{3}+3k^{2}+3k+1) - (8k^{3}+12k^{2}+6k+1) = 6k(2k+3)+7$				
	$\Rightarrow which is odd$				
	Hence odd for all $n \in \square$ *	A1*	2.4		
		(4	marks)		
	Notes	(.	inui K5)		
	General guidance				
	It is likely that you will see a mixture of methods and approaches within some so	o <mark>lutions.</mark>			
T T1 1	Mark the approach which scores the highest number of marks.				
There s	hould be no errors in the algebra but allow e.g. invisible brackets to be "recovered". d the final mark if n is used instead of k or reference to $n \in \Box$ but $n \in \Box^+$ is accortable	10			
vv iunio	If the final mark if <i>n</i> is used instead of <i>k</i> of reference to $n \in \square$ but $n \in \square$ is acceptate	ne.			
Main s	wheme algebraic method using e.g. $n = 2k$ and $n = 2k \pm 1$				
You wi	l need to look at both cases and mark the one which is fully correct first.				
Allow a	different variable to k and may be different letters for odd and even				
	(1,1)	.• ,			
MI:	or the key step attempting to find $(n+1)^n - n^n$ when $n = 2k$ or $n = 2k \pm 1$ and attempting to				
	nultiply out and simplify to achieve a three term quadratic (allow equivalent represence $g_{1} = 2k + 2$ or $2n + 5$)	itation of	odd or		
	Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$				
A1:	Complete argument for $n = 2k$ or $n = 2k+1$ (or e.g. $n = 2k-1$) showing the result	is odd.			
	Correct simplified quadratic expression e.g. $12k^2 + 6k + 1$ (when $n = 2k$) $12k^2 + 18k$	± 7 (whe	n		
	$n = 2k + 1$). $12k^2 - 6k + 1$ (when $n = 2k - 1$) (may be factorised)		-11		
•	A reason why the expression is odd e.g. $2k(6k+3)+1$ or may use a divisibility argum	nent e.g.			
	$\frac{12k^2 + 6k + 1}{2} = 6k^2 + 3k + \frac{1}{2}$	C			
	2 2 Concludes "odd" a.e. (may be within their final conclusion)				
	There should be no errors in the algebra but allow e.g. invisible brackets if they are "	recovered	1"		
	Condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$				
dM1:	Attempts to find $(n+1)^3 - n^3$ when $n = 2k$ and $n = 2k \pm 1$ and attempts to multiply of	out and si	mplify		
	o achieve a three term quadratic (allow equivalent representation of odd or even e.g. $2n \pm 5$	n=2k-	+2,		
	Condone arithmetical slips and condone the use of e.g. $n = 2n$ and $n = 2n \pm 1$				

A1*: Complete argument for **both** n = 2k and n = 2k+1 (or e.g. n = 2k-1) showing the result is odd for all $n \in \mathbb{D}$)

Requires for both cases:

- Correct simplified expressions for both odd and even (which may be factorised)
- A reason why both of the expressions are odd
- Minimal conclusion (may be within their final conclusion)

An overall conclusion is also required. "Hence odd for all $n (\in \Box)$ " Accept "hence proven", "statement proved", "QED"

The conclusion for when n = 2k and n = 2k+1 may be within the final conclusion rather than separate which is acceptable e.g. "when n = 2k and when n = 2k+1 the expression is odd, hence proven" (following correct simplified expressions and reasons)

	$(n+1)^3$	n^3	$(n+1)^3 - n^3$
n = 2k - 1	$8k^3$	$8k^3 - 12k^2 + 6k - 1$	$12k^2 - 6k + 1$
n = 2k	$8k^3 + 12k^2 + 6k + 1$	$8k^{3}$	$12k^2 + 6k + 1$
n = 2k + 1	$8k^3 + 24k^2 + 24k + 8$	$8k^3 + 12k^2 + 6k + 1$	$12k^2 + 18k + 7$

Alternative methods:

Algebraic with logic example

- M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic. Condone arithmetical slips.
- A1: Correct quadratic expression $3n^2 + 3n + 1$
- dM1: Attempts to factorise their quadratic such that $n^2 + n \rightarrow n(n+1)$ within their expression e.g. 3n(n+1)+1
- A1*: Explains that e.g. n(n+1) is always even as it is the product of two consecutive numbers so

3n(n+1) is odd \times even = even so 3n(n+1)+1 is odd hence odd for all $n \in \square$

Proof by contradiction example

- M1: Attempts to multiply out the brackets and simplifies to achieve a three term quadratic.
- A1: Correct quadratic expression $3n^2 + 3n + 1$
- dM1: Sets $3n^2 + 3n + 1 = 2k$ (for some integer k) $\Rightarrow 3n(n+1) = 2k 1$

A1*: Explains that n(n+1) is always even as it is the product of two consecutive numbers so 3n(n+1) is odd × even = even but 2k-1 is odd hence we have a contradiction so $(n+1)^3 - n^3$ is odd (for all $n (\in \square)$). There must have been a correct opening statement setting up the contradiction e.g. "assume that there exists a value for *n* for which $(n+1)^3 - n^3$ is even"

Solutions via just logic (no algebraic manipulation)

e.g.

If *n* is odd, then $(n+1)^3 - n^3$ is even³ - odd³ = even - odd = odd

If *n* is even, then $(n+1)^3 - n^3$ is $odd^3 - even^3 = odd - even = odd$

Both cases must be considered to score any marks and scores SC 1010 if fully correct

Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for n = k, substitutes n = k + 1 into $(n + 1)^3 - n^3$, multiplies out the brackets and attempts to simplify to a three term quadratic e.g. $3k^2 + 9k + 7$ Condone arithmetical slips

A1:
$$(f(k+1) = 3k^2 + 3k + 1 + 6(k+1) =) (k+1)^3 - k^3 + 6(k+1) = f(k) + 6(k+1)$$

which is odd + even = odd

dM1: Attempts to substitute $n = 1 \implies (1+1)^3 - 1^3 = 7$ (which is true) (Condone arithmetical slips evaluating)

- A1*: Explains that
 - it is true when n = 1
 - if it is true for n = k then it is true for n = k + 1
 - therefore it is true for all $n \in \square$

Question	Scheme	Marks	AOs			
15(a)	$\dots xe^x + \dots e^x$	M1	1.1b			
	$k(xe^{x}+e^{x})$	A1	1.1b			
	$\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right) = \frac{1}{2} \times 3e^{3x} \left(e^{3x}-2\right)^{-\frac{1}{2}}$	B1	1.1b			
	$(f'(x) =) \frac{(e^{3x} - 2)^{\frac{1}{2}} ("7"xe^{x} + "7"e^{x}) - "\frac{3}{2}"e^{3x} (e^{3x} - 2)^{-\frac{1}{2}} \times "7"xe^{x}}{e^{3x} - 2}$	dM1	2.1			
	$f'(x) = \frac{7e^{x} (e^{3x}(2-x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}}$	A1	1.1b			
		(5)				
(b)	$e^{3x}(2-x)-4x-4=0 \Rightarrow x(e^{3x}\pm)=e^{3x}\pm$	M1	1.1b			
	$\Rightarrow x = \frac{2e^{3x} - 4}{e^{3x} + 4} *$	A1*	2.1			
		(2)				
(c)	Draws a vertical line $x = 1$ up to the curve then across to the line $y = x$ then up to the curve finishing at the root (need to see a minimum of 2	B1	2.1			
	vertical and horizontal lines tending to the root)	(1)				
(d)(i)	2^{-3} 4	(1)				
()(-)	$x_2 = \frac{2e^2 - 4}{e^3 + 4} = 1.5017756$	M1	1.1b			
	$x_2 = $ awrt 1.502	A1	1.1b			
(ii)	$\beta = 1.968$	dB1	2.2b			
		(3)				
(e)	$h(x) = \frac{2e^{3x} - 4}{e^{3x} + 4} - x$ h(0.4315) = -0.000297 h(0.4325) = 0.000947	M1	3.1a			
	 Both calculations correct and e.g. states: There is a change of sign e.g f'(x) is continuous α = 0.432 (to 3dp) 	A1cao	2.4			
		(2)				
		(13	marks)			
	Notes					
(a) M1: Attempts the product rule on xe^x (or may be $7xe^x$) achieving an expression of the form $xe^x \pme^x$. If it is clear that the quotient rule has been applied instead which may be quoted then M0.						
A1: $k(xe^{x} + e^{x})$ (e.g. $7(xe^{x} + e^{x})$) or equivalent which may be unsimplified (may be implied by further work)						
D1 (d	$\left(\left(\frac{1}{3^{3x} - 2} \right) \right)^{\frac{1}{2}} = 2^{\frac{3x}{3}} \left(\frac{3x}{3^{3x} - 2} \right)^{-\frac{1}{2}} \left(\frac{1}{3^{3x} - 1} \right)^{\frac{1}{2}} \left(\frac{1}{3^{3x} - 1} \right)^{\frac{1}{2}}$					

B1:
$$\left(\frac{d}{dx}\left(\sqrt{e^{3x}-2}\right)\right) = \frac{1}{2} \times 3e^{3x}\left(e^{3x}-2\right)^{-\frac{1}{2}}$$
 (simplified or unsimplified)

dM1: Attempts to use the quotient rule. It is dependent on the previous method mark.
Score for achieving an expression of the form
$$(f'(x) =) \frac{(e^{3x} - 2)^{\frac{2}{2}}("7" xe^{x} + "7" e^{x}) - \frac{2}{2} e^{3x}(e^{3x} - 2)^{\frac{1}{2}} x"7" xe^{x}}{e^{x} - 2} \text{ or equivalent (do not be concerned by the constants for their "7" or their " $\frac{3}{2}$ " which may be both 1)

If it is clear that the quotient rule has been applied the wrong way round then score M0. Alternatively, applies the product rule. Score for achieving an expression of the form
$$(f'(x) =) \frac{(e^{3x} - 2)^{\frac{1}{2}}("7" xe^{x} + "7" e^{x}) - u^{\frac{3}{2}} e^{3x}(e^{3x} - 2)^{\frac{2}{2}} x"7" xe^{x} or equivalent (do not be concerned by the constants for their "7" or their " $\frac{3}{2}$ " which may be both 1)

Do not condone invisible brackets.

A1:
$$(f'(x) =) \frac{7e^{x}(e^{3x}(-2x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}} following a fully correct differentiated expression.

You may need to check to see if (a) is continued after other parts for evidence of this.

Condone the lack of f'(x) = on the left hand side or allow the use of $\frac{dy}{dx}$ or y' instead.

Alternative (a) attempt using the triple product rule

e.g. $\frac{d}{dx} \left(7xe^{x}(e^{3x} - 2)^{\frac{1}{2}} \right) = 7e^{x}(e^{3x} - 2)^{\frac{1}{2}} + 7xe^{x}(e^{3x} - 2)^{\frac{1}{2}} + 3xe^{3x}(e^{3x} - 2)^{\frac{3}{2}}$

M1: Attempts the product rule on $xe^{x} \to ...xe^{x} \pm ...e^{x}$ which may be scen within the expression

 $...e^{x}(e^{3x} - 2)^{\frac{1}{2}} \pm ...xe^{x}(e^{3x} - 2)^{\frac{1}{2}} + ...xe^{x}(e^{3x} - 2)^{\frac{1}{2}} + ...xe^{x}(e^{3x} - 2)^{\frac{1}{2}} + ...xe^{3}(e^{3x} - 2)^{\frac{1}{2}} + ...xe^{3x}(e^{3x} - 2)^{\frac{1}{2}} + ...xe^{3x}($$$$$$$

A1*:	Achieves the given answer with no errors including invisible brackets. If they do not reach the					
	printed answer then it is A0. If they subsequently write $x_{n+1} = \frac{2e^{3x_n} - 4}{3x_{n+1}}$ then isw					
(a)	$e^{3x_n} + 4$					
B1:	Starting at $x = 1$ look for at least 2 sets of vertical and horizontal lines drawn (may be dashes)					
211	tending to β . Condone a lack of arrows on the lines but the sequence of lines should finish at the					
	point of intersection where the root is. Condone the initial vertical line not starting from the x-					
	axis. Mark the intention to draw horizontal and vertical lines. If they have any lines to the left of					
	x = 1 this is B0. If they use both diagrams and do not indicate which one they want marking, then the					
	"copy of Diagram 1" should be marked.					
	Examples scoring B1: Examples scoring B0:					
(d)(i)	Substitutes 1 into the iterative formula. The values anthodded in the formula is sufficient for this					
1011:	mark May be implied by awrt 1 50					
A1:	awrt 1.502 isw					
(d)(ii)						
dB1:	1.968 cao (which can only be scored if M1 is scored in (d)(i))					
SC:	If (d)(1) is rounded to 1.50 then allow 1.97 in (d)(11) to score M1A0dB1 for (d)					
M1:	Attempts to substitute $x = 0.4315$ and 0.4325 into a suitable function and gets one value correct (rounded or truncated to 1sf). It is allowable to use a tighter interval that contains the root 0.4317388728 If no function is stated then may be implied by their answers to e.g. f'(0.4315), f'(0.4325) You will need to check their calculation is correct.					
	Other possible functions include: $2e^{3x} = 4$					
•	$h(x) = x - \frac{2e^{-4}}{e^{3x} + 4}$ (other way round to MS) $h(0.4315) = 0.0002974$, $h(0.4325) = -0.0009479$					
•	their f'(x) = $\pm \left(\frac{7e^x (e^{3x} (2-x) - 4x - 4)}{2(e^{3x} - 2)^{\frac{3}{2}}} \right)$					
	(If correct A and B then $f'(0.4315) = \mp 0.005789$, $f'(0.4325) = \pm 0.01831$)					
•	their $g(x) = \pm (e^{3x}(2-x) - 4x - 4)$					
	(If correct A and B then $g(0.4315) = \mp 0.002275, g(0.4325) = \pm 0.007261)$					
A1:	Requires					
	• Both calculations correct (rounded or truncated to 1sf)					
	• A statement that there is a change in sign and that their function is continuous (must refer to the function used for the substitution (which is not $f(x)$)					
	Accept equivalent statements for $f'(0.4315) < 0$, $f'(0.4325) > 0$ e.g.					
	$f'(0.4315) \times f'(0.4325) < 0$, "one negative one positive". A minimum is "change of sign and					
	continuous" but do not allow this mark if the comment about continuity is clearly incorrect					
	e.g. "because x is continuous" or "because the interval is continuous"					
	• A minimal conclusion e.g. "hence $\alpha = 0.432$ ", "so rounds to 0.432". Do not allow "hence root"					